# Polarizing mechanisms for stored $p$ and $\bar{p}$ beams interacting with a polarized target

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The kinetics of the polarization buildup during the interaction of stored protons (antiprotons) with a polarized target is considered. It is demonstrated that for small scattering angles, when a projectile remains in the beam, the polarization buildup is completely due to the spin-flip transitions. The corresponding cross sections turn out to be negligibly small for a hydrogen gas target as well as for a pure electron target. For the latter, the filtering mechanism also does not provide a noticeable beam polarization.

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## I. INTRODUCTION

Now that the use of a polarized hydrogen gas target to polarize stored antiprotons is under discussion for experiments at the GSI laboratory in Germany (see, Ref. [1] and literature therein), we look again at earlier theoretical work dealing with modifications to the method first proposed in Ref. [2]. In Ref. [2], it was proposed that the dependence of the scattering cross section on the orientations of the target and projectile proton spins be exploited. This dependence would give rise to a filtering mechanism whereby protons of a beam with positive spin projections in the direction of the target polarization would scatter out of the beam at a rate different from protons with a negative spin projection. As a result, the beam would become polarized.

The method suggested in Ref. [2] has been realized in the experiment [3], where 23 MeV stored protons scatter on an internal gas target of polarized hydrogen atoms. After 90 minutes, the intensity of the beam was 5% of the initial one and the polarization degree amounted to 2.4%. In Ref. [3], the rate of polarization buildup due to the filtering mechanism was also estimated theoretically, taking into account only the strong $pp$ interaction. This estimate is noticeably different from experimental results. The explanation of this disagreement is proposed in Ref. [4]. In that paper the importance of the interference of the Coulomb amplitude and the spin-dependent part of the hadronic amplitude is emphasized. Under conditions of the experiment [3], this effect diminishes the corresponding cross section by more than 40% thereby considerably improving agreement with the experiment. Our calculations confirm the estimate of this effect obtained in Ref. [4].

The remaining difference between the experimental and theoretical results, which is rather small, is explained in Ref. [4] by two new mechanisms. Both mechanisms are related to scattering at an angle $\vartheta$ smaller than the acceptance angle $\theta_{\rm acc} \ll 1$, where protons remain in the beam. The first effect, suggested in Ref. [5], is due to the interaction of a projectile with polarized electrons of the hydrogen gas target. The second one, considered in Ref. [4], is due to scattering off polarized protons in the target at $\vartheta < \theta_{\rm acc}$. The estimate of these two effects, made in Ref. [4], gives contributions to the rate of polarization buildup which are similar in absolute values but of opposite signs so that their sum is small. The magnitude of each effect is comparable to that corresponding to filtering effect. The result accounting for all three contributions agrees very well with the experiment. Thus as a result of the predictions in Ref. [5], it has been proposed in Ref. [1] to polarize an antiproton beam by using a hydrogen gas target with a high electron polarization and a low proton polarization.

In the present paper, we demonstrate that the consideration of both new effects performed in Refs. [4,5] is not correct. For scattering at $\vartheta < \theta_{\rm acc}$ (the projectile remains in the beam), the polarization buildup is completely due to spin-flip transitions for the projectiles. For $\vartheta \ll 1$, a noticeable contribution to the spin-dependent part of the hadronic differential cross section appears as a result of the interference between the spin-independent part of a Coulomb amplitude and the spin-dependent part of the strong amplitude. This is because of the singularity of the Coulomb term at small scattering angles ($\propto 1/\vartheta^2$). The spin-dependent part of the nonrelativistic Coulomb amplitude appears due to the identity of protons. For spin-flip transitions at $\vartheta \ll 1$, this amplitude has no singularity. Thus the interference between the Coulomb and the strong part of the amplitude is almost absent for spin-flip transitions, and the corresponding cross section is negligibly small. Taking into account relativistic corrections do not change this conclusion. The formulas used in Refs. [4,5] for the description of these two effects (where the projectiles remain in the beam) correspond to such an interference and are, therefore, irrelevant to the kinetics of polarization.

## II. KINETICS OF POLARIZATION

We consider a beam of particles with the densities in momentum space $f_+(\boldsymbol{p},t)$ and $f_-(\boldsymbol{p},t)$, where subscripts correspond to the spin projections $\pm 1/2$ on a quantization axis. Let $W_{s_f s_i}(\boldsymbol{p}_f,\boldsymbol{p}_i)$ be the probability of the transition from a state with the momentum $\boldsymbol{p}_i$ and polarization $s_i$ to a state with the momentum $\boldsymbol{p}_f$ and polarization $s_f$. Note that the probability $W_{s_f s_i}(\boldsymbol{p}_f,\boldsymbol{p}_i)$ depends on the polarization of the target. We

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then have the conventional kinetic equation for the densities $f_{\pm}$,

$$\frac{\partial}{\partial t} f_{+}(\boldsymbol{p},t) = - \int d\boldsymbol{p}' [W_{++}(\boldsymbol{p}',\boldsymbol{p}) + W_{-+}(\boldsymbol{p}',\boldsymbol{p})] f_{+}(\boldsymbol{p},t) + \int_{\boldsymbol{p}' \in \Gamma} d\boldsymbol{p}' [W_{++}(\boldsymbol{p},\boldsymbol{p}') f_{+}(\boldsymbol{p}',t) + W_{+-}(\boldsymbol{p},\boldsymbol{p}') f_{-}(\boldsymbol{p}',t)],$$

$$\frac{\partial}{\partial t} f_{-}(\boldsymbol{p},t) = - \int d\boldsymbol{p}' [W_{--}(\boldsymbol{p}',\boldsymbol{p}) + W_{+-}(\boldsymbol{p}',\boldsymbol{p})] f_{-}(\boldsymbol{p},t) + \int_{\boldsymbol{p}' \in \Gamma} d\boldsymbol{p}' [W_{-+}(\boldsymbol{p},\boldsymbol{p}') f_{+}(\boldsymbol{p}',t) + W_{--}(\boldsymbol{p},\boldsymbol{p}') f_{-}(\boldsymbol{p}',t)]. \quad (1)$$

Here $\boldsymbol{p}' \in \Gamma$ means that the momentum $\boldsymbol{p}'$ belongs to the beam momentum space (the angle between the momentum $\boldsymbol{p}'$ and the beam axis is less than $\theta_{\mathrm{acc}}$). Taking the integral over $\boldsymbol{p} \in \Gamma$ in Eq. (1), we obtain

$$\begin{aligned}\frac{\partial}{\partial t}\int_{\boldsymbol{p}\in\Gamma} d\boldsymbol{p}[f_{+}(\boldsymbol{p},t) - f_{-}(\boldsymbol{p},t)] &= -\int_{\boldsymbol{p}'\notin\Gamma} d\boldsymbol{p}' \int_{\boldsymbol{p}\in\Gamma} d\boldsymbol{p}[W_{++}(\boldsymbol{p}',\boldsymbol{p}) + W_{-+}(\boldsymbol{p}',\boldsymbol{p})] f_{+}(\boldsymbol{p},t) \\ &+ \int_{\boldsymbol{p}'\notin\Gamma} d\boldsymbol{p}' \int_{\boldsymbol{p}\in\Gamma} d\boldsymbol{p}[W_{--}(\boldsymbol{p}',\boldsymbol{p}) + W_{+-}(\boldsymbol{p}',\boldsymbol{p})] f_{-}(\boldsymbol{p},t) \\ &+ 2\int_{\boldsymbol{p}'\in\Gamma} d\boldsymbol{p}' \int_{\boldsymbol{p}\in\Gamma} d\boldsymbol{p}[W_{+-}(\boldsymbol{p}',\boldsymbol{p}) f_{-}(\boldsymbol{p},t) - W_{-+}(\boldsymbol{p}',\boldsymbol{p}) f_{+}(\boldsymbol{p},t)],\end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial t}\int_{\boldsymbol{p}\in\Gamma} d\boldsymbol{p}[f_{+}(\boldsymbol{p},t) + f_{-}(\boldsymbol{p},t)] &= -\int_{\boldsymbol{p}'\notin\Gamma} d\boldsymbol{p}' \int_{\boldsymbol{p}\in\Gamma} d\boldsymbol{p}[W_{++}(\boldsymbol{p}',\boldsymbol{p}) + W_{-+}(\boldsymbol{p}',\boldsymbol{p})] f_{+}(\boldsymbol{p},t) \\ &- \int_{\boldsymbol{p}'\notin\Gamma} d\boldsymbol{p}' \int_{\boldsymbol{p}\in\Gamma} d\boldsymbol{p}[W_{--}(\boldsymbol{p}',\boldsymbol{p}) + W_{+-}(\boldsymbol{p}',\boldsymbol{p})] f_{-}(\boldsymbol{p},t). \quad (2)\end{aligned}$$

As expected, the terms in Eq. (2) in which both momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ belong to $\Gamma$ contain only spin-flip probabilities. In other words, scattering without loss of particles may lead to a beam polarization solely due to spin-flip transitions. Due to phase space cooling, the distributions $f_{\sigma}(\boldsymbol{p},t)$ are peaked in the narrow region around a momentum $\boldsymbol{p}_0$. Assuming that the probabilities $W_{s_f s_i}(\boldsymbol{p}_f,\boldsymbol{p}_i)$ change very little across the small momentum range of the beam, we obtain

$$\frac{d}{dt}[N_{+}(t) - N_{-}(t)] = -\Omega_{+}^{\mathrm{out}} N_{+}(t) + \Omega_{-}^{\mathrm{out}} N_{-}(t) + 2[\Omega_{+-} N_{-}(t) - \Omega_{-+} N_{+}(t)],$$

$$\frac{d}{dt}[N_{+}(t) + N_{-}(t)] = -[\Omega_{+}^{\mathrm{out}} N_{+}(t) + \Omega_{-}^{\mathrm{out}} N_{-}(t)]. \quad (3)$$

Here

$$\Omega_{+}^{\mathrm{out}} = \int_{\boldsymbol{p}'\notin\Gamma} d\boldsymbol{p}'[W_{++}(\boldsymbol{p}',\boldsymbol{p}_0) + W_{-+}(\boldsymbol{p}',\boldsymbol{p}_0)], \quad \Omega_{-}^{\mathrm{out}} = \int_{\boldsymbol{p}'\notin\Gamma} d\boldsymbol{p}'[W_{--}(\boldsymbol{p}',\boldsymbol{p}_0) + W_{+-}(\boldsymbol{p}',\boldsymbol{p}_0)],$$

$$\Omega_{+-} = \int_{\boldsymbol{p}'\in\Gamma} d\boldsymbol{p}' W_{+-}(\boldsymbol{p}',\boldsymbol{p}_0), \quad \Omega_{-+} = \int_{\boldsymbol{p}'\in\Gamma} d\boldsymbol{p}' W_{-+}(\boldsymbol{p}',\boldsymbol{p}_0), \quad N_{\pm}(t) = \int_{\boldsymbol{p}\in\Gamma} d\boldsymbol{p} f_{\pm}(\boldsymbol{p},t). \quad (4)$$

The solution to Eq. (3), with the initial condition $N_{+}(0) = N_{-}(0) = N_0/2$, reads

$$N(t) = N_{+}(t) + N_{-}(t) = N_0\left(\cosh(\Omega t) + \frac{\Omega_{+-} + \Omega_{-+}}{2\Omega}\sinh(\Omega t)\right)\exp(-\Omega_{\mathrm{tot}} t),$$

$$P_B(t) = \frac{N_{+}(t) - N_{-}(t)}{N_{+}(t) + N_{-}(t)} = \frac{\Omega_{+-} - \Omega_{-+} + \frac{1}{2}(\Omega_{-}^{\mathrm{out}} - \Omega_{+}^{\mathrm{out}})}{\Omega + \frac{1}{2}(\Omega_{+-} + \Omega_{-+})\tanh(\Omega t)}\tanh(\Omega t), \quad (5)$$

where $N(t)$ is the total number of particles in the beam, $P_B(t)$ is the beam polarization, and

$$\Omega_{\mathrm{tot}} = \tfrac{1}{2}(\Omega_{+}^{\mathrm{out}} + \Omega_{-}^{\mathrm{out}} + \Omega_{+-} + \Omega_{-+}),$$

$$\Omega = \tfrac{1}{2}[(\Omega_{-}^{\mathrm{out}} - \Omega_{+}^{\mathrm{out}})^2 + (\Omega_{+-} + \Omega_{-+})^2 + 2(\Omega_{-}^{\mathrm{out}} - \Omega_{+}^{\mathrm{out}})(\Omega_{+-} - \Omega_{-+})]^{1/2}. \quad (6)$$

For the scattering angle $\vartheta > \theta_{\mathrm{acc}}$, the momentum transfer is much larger than $1/a_0$ ($a_0$ is the Bohr radius, $\hbar = c = 1$). In this case scattering off a hydrogen atom can be considered as independent scattering off a free electron and a free proton. The maximum scattering angle of a proton off an electron at rest is less than $\theta_{\mathrm{acc}}$ for any storage ring. Therefore, scattering off electrons at rest does not contribute to $\Omega_{\pm}^{\mathrm{out}}$. If the protons in the hydrogen target are unpolarized (as in the scheme considered in Ref. [1]), then $\Omega_{-}^{\mathrm{out}} = \Omega_{+}^{\mathrm{out}}$ and we have from Eq. (5)

$$P_B(t) = \frac{\Omega_{+-} - \Omega_{-+}}{\Omega_{+-} + \Omega_{-+}}[1 - \exp[-(\Omega_{+-} + \Omega_{-+})t]]. \quad (7)$$

As shown in the next section, for a noticeable proton polarization of the target, $|\Omega_-^{\rm out} - \Omega_+^{\rm out}|$ is much larger than $\Omega_{+-}$ and $\Omega_{-+}$. In this case

$$P_B(t) = \tanh\left(\frac{t}{2}(\Omega_-^{\rm out} - \Omega_+^{\rm out})\right). \quad (8)$$

## III. PROBABILITIES AND CROSS SECTIONS FOR $pp$ SCATTERING

Let us direct the polar axis along the unit vector $\boldsymbol{\nu} = \boldsymbol{p}_0/p_0$. Then the cross section of $pp$ scattering integrated over the azimuth angle and summed up over final spin states of both protons in the center-of-mass frame reads

$$d\sigma = 2\pi \sin\vartheta d\vartheta\{F_0(\vartheta) + (\boldsymbol{\zeta}_t \cdot \boldsymbol{\zeta}_b)F_1(\vartheta) + (\boldsymbol{\zeta}_t \cdot \boldsymbol{\nu})(\boldsymbol{\zeta}_b \cdot \boldsymbol{\nu}) \times [F_2(\vartheta) - F_1(\vartheta)]\}, \quad (9)$$

where $\boldsymbol{\zeta}_t$ and $\boldsymbol{\zeta}_b$ are the unit polarization vectors of the protons from the target and the beam, respectively. The functions $F_0(\vartheta)$, $F_1(\vartheta)$, and $F_2(\vartheta)$ are

$$F_0(\vartheta) = I_{0000} = \tfrac{1}{2}(|M_1|^2 + |M_2|^2 + |M_3|^2 + |M_4|^2 + 4|M_5|^2),$$

$$F_1(\vartheta) = \tfrac{1}{2}I_{0000}[A_{00nn} + \cos^2(\vartheta/2)A_{00mm} + \sin^2(\vartheta/2)A_{00ll} + \sin(\vartheta)A_{00ml}] = |M_5|^2 + \mathrm{Re}(M_1M_2^*),$$

$$F_2(\vartheta) = I_{0000}[\sin^2(\vartheta/2)A_{00mm} + \cos^2(\vartheta/2)A_{00ll} - \sin(\vartheta)A_{00ml}] = \tfrac{1}{2}[|M_3|^2 + |M_4|^2 - |M_1|^2 - |M_2|^2]. \quad (10)$$

Here the observables $I_{0000}$, $A_{00mm}$, $A_{00nn}$, $A_{00ll}$, $A_{00ml}$ and the helicity amplitudes $M_i$ are defined as in Ref. [6].

Let $\boldsymbol{P}_T$ be the target polarization vector, and $\boldsymbol{\zeta}_T = \boldsymbol{P}_T/P_T$. We direct the quantization axis along the unit vector $\boldsymbol{\zeta}_T$. Averaging the cross section in Eq. (9) over particles in the target, we obtain for the quantities $\Omega_\pm^{\rm out}$

$$\Omega_\pm^{\rm out} = nf\{\sigma_0 \pm P_T[\sigma_1 + (\boldsymbol{\zeta}_T \cdot \boldsymbol{\nu})^2(\sigma_2 - \sigma_1)]\},$$

$$\sigma_i = 2\pi\int_{\theta_{\rm acc}}^{\pi/2} d\vartheta \sin\vartheta\, F_i(\vartheta). \quad (11)$$

Here $n$ is the areal density of the target, $f$ is a revolution frequency, $\theta_{\rm acc}$ is defined in the center-of-mass frame. The function $P_B(t)$ in Eq. (5) contains $\Omega_\pm^{\rm out}$ only in the combination $\Omega_+^{\rm out} - \Omega_-^{\rm out}$. For $|\sigma_2| > |\sigma_1|$, this difference is maximal at $\boldsymbol{\zeta}_T \parallel \boldsymbol{\nu}$. For $|\sigma_2| < |\sigma_1|$, the difference is maximal at $\boldsymbol{\zeta}_T \perp \boldsymbol{\nu}$. In Eqs. (5) and (6), the quantities $\Omega_{+-}$ and $\Omega_{-+}$, which are related to the spin-flip transitions, have the form

$$\Omega_{-+} + \Omega_{+-} = nf[\sigma_{2s} + (\boldsymbol{\zeta}_T \cdot \boldsymbol{\nu})^2(\sigma_{1s} - \sigma_{2s})],$$

$$\Omega_{-+} - \Omega_{+-} = nfP_T\{\sigma_{2d} + (\boldsymbol{\zeta}_T \cdot \boldsymbol{\nu})^2(\sigma_{1d} - \sigma_{2d})\},$$

$$\sigma_X = 2\pi\int_0^{\theta_{\rm acc}} d\vartheta \sin\vartheta G_X(\vartheta). \quad (12)$$

In terms of the helicity amplitudes, the functions $G_X(\vartheta)$ are

$$G_{1s}(\vartheta) = \tfrac{1}{2}[|2\cos(\vartheta/2)M_5 + \sin(\vartheta/2)(M_1 + M_3)|^2 + |2\sin(\vartheta/2)M_5 + \cos(\vartheta/2)(M_4 - M_2)|^2 + \sin^2(\vartheta/2) \times |M_1 - M_3|^2 + \cos^2(\vartheta/2)|M_2 + M_4|^2],$$

$$G_{2s}(\vartheta) = \tfrac{1}{2}[G_{1s} + \cos^2(\vartheta/2)|M_1 - M_3|^2 + \sin^2(\vartheta/2)|M_2 + M_4|^2],$$

$$G_{1d}(\vartheta) = \sin\vartheta\, \mathrm{Re}[M_5^*(M_2 + M_4 + M_3 - M_1)] + \sin^2(\vartheta/2) \times (|M_3|^2 - |M_1|^2) + \cos^2(\vartheta/2)(|M_4|^2 - |M_2|^2),$$

$$G_{2d}(\vartheta) = \mathrm{Re}\left\{\tfrac{1}{2}\sin\vartheta[M_5^*(M_2 + M_4 + M_3 - M_1)] + M_2^*(M_1 - M_3) + \sin^2(\vartheta/2)(M_4^*M_1 + M_2^*M_3)\right\}. \quad (13)$$

Each hadronic amplitude $M_i$ can be represented as a sum $M_i = M_i^{\rm em} + M_i^h$ of a pure electromagnetic amplitude, $M_i^{\rm em}$, and the strong amplitude $M_i^h$. We emphasize that the amplitude $M_i^h$ does not coincide with the strong amplitude calculated without account for the electromagnetic interaction. The amplitudes $M_i^h$ are not singular at small scattering angle $\vartheta$. More precisely, at $\vartheta \to 0$, $M_{1,2,3}^h$ are nonzero constant, $M_4^h \propto \vartheta^2$, and $M_5^h \propto \vartheta$. In the nonrelativistic limit, the amplitudes $M_i^{\rm em}$ pass into the amplitude $M_i^C$, which are the matrix element of the operator $\hat{M}^C$in spin space [7]

$$\hat{M}^C = f(\vartheta) - \tfrac{1}{2}(1 + \boldsymbol{\sigma}_b \cdot \boldsymbol{\sigma}_t)f(\pi - \vartheta),$$

$$f(\vartheta) = -\frac{\alpha}{4vp\sin^2(\vartheta/2)}\exp\{-i(\alpha/v)\ln[\sin(\vartheta/2)]\}, \quad (14)$$

where $v = p/m_p$ is the proton velocity in the center-of-mass frame, $\boldsymbol{\sigma}$ are the Pauli matrices, $\alpha$ is the fine structure constant. The presence of the spin operators in $\hat{M}^C$ is completely due to the identity of protons. From Eq. (14), we obtain for $M_i^C$,

$$M_1^C = \cos^2(\vartheta/2)f(\vartheta) + \sin^2(\vartheta/2)f(\pi - \vartheta),$$

$$M_2^C = -[\sin^2(\vartheta/2)f(\vartheta) + \cos^2(\vartheta/2)f(\pi - \vartheta)],$$

$$M_3^C = \cos^2(\vartheta/2)[f(\vartheta) - f(\pi - \vartheta)],$$

$$M_4^C = \sin^2(\vartheta/2)[f(\vartheta) - f(\pi - \vartheta)],$$

$$M_5^C = -\tfrac{1}{2}\sin\vartheta[f(\vartheta) - f(\pi - \vartheta)]. \quad (15)$$

Using Eq. (15), we can consider the interrelation between the electromagnetic and strong contributions to $\Omega_\pm^{\rm out}$. If $\theta_{\rm acc} \ll \alpha/(vpH)$, $H$ is the typical magnitude of the strong amplitudes, then the main contribution to the cross section $\sigma_0$ in

Eq. (11) comes from the integration region $\vartheta \sim \theta_{acc} \ll 1$, where $F_0(\vartheta) \simeq |f(\vartheta)|^2 \propto 1/\vartheta^4$. Thus, this contribution has an overwhelmingly electromagnetic origin,

$$\sigma_0 \approx \sigma_0^C = \pi\alpha^2/(vp\theta_{acc})^2. \quad (16)$$

For the electromagnetic part of the functions $F_{1,2}$, we have $F_1^C = F_2^C = -\mathrm{Re}[f^*(\vartheta)f(\pi-\vartheta)]$. For $\vartheta \ll 1$, we have $F_1^C = F_2^C \propto 1/\vartheta^2$. The corresponding contribution to $\sigma_{1,2}$ reads

$$\sigma_1^C = \sigma_2^C = -\frac{\pi\alpha}{2vp^2}\sin\Psi, \quad \Psi = \frac{\alpha}{v}\ln(2/\theta_{acc}). \quad (17)$$

The interference terms $\sigma_i^{\mathrm{int}}$ in $\sigma_i$ can be estimated with logarithmic accuracy as

$$\sigma_0^{\mathrm{int}} = -\frac{2\pi}{p}\{\sin\Psi\,\mathrm{Re}[M_3^h(0)+M_1^h(0)] + (1 - \cos\Psi)\mathrm{Im}[M_3^h(0)+M_1^h(0)]\},$$

$$\sigma_1^{\mathrm{int}} = -\frac{2\pi}{p}[\sin\Psi\,\mathrm{Re}\,M_2^h(0) + (1-\cos\Psi)\mathrm{Im}\,M_2^h(0)],$$

$$\sigma_2^{\mathrm{int}} = -\frac{2\pi}{p}\{\sin\Psi\,\mathrm{Re}[M_3^h(0)-M_1^h(0)] + (1 - \cos\Psi)\,\mathrm{Im}[M_3^h(0)-M_1^h(0)]\}. \quad (18)$$

We illustrate the scale of different contributions, giving their numerical values corresponding to the parameters of the experiment [3], $E_{\mathrm{lab}}=23$ MeV, $\theta_{acc}=8.8$ mrad. Using the data base [8] for the strong amplitudes $M_i^h$, we obtain

$$\sigma_0 = 6444 \text{ mb}, \quad \sigma_0^{\mathrm{int}} = -56 \text{ mb}, \quad \sigma_0^C = 6357 \text{ mb},$$

$$\sigma_1 = -89 \text{ mb}, \quad \sigma_1^{\mathrm{int}} = 39 \text{ mb}, \quad \sigma_1^C = -1 \text{ mb},$$

$$\sigma_2 = -66 \text{ mb}, \quad \sigma_2^{\mathrm{int}} = 66 \text{ mb}, \quad \sigma_2^C = -1 \text{ mb}. \quad (19)$$

Recollect that $\sigma_i = \sigma_i^{\mathrm{em}} + \sigma_i^{\mathrm{int}} + \sigma_i^h$ and $\sigma_i^{\mathrm{em}} \simeq \sigma_i^C$ in the nonrelativistic case. Note that the numbers obtained from Eqs. (16)–(18) are in good agreement with the ones in Eq. (19). The role of the interference of the strong and electromagnetic amplitudes is additionally illustrated in Fig. 1. In this figure, the function $2\pi\sin\vartheta F_1(\vartheta)$ calculated using the full amplitudes $M_i = M_i^{\mathrm{em}} + M_i^h$ (solid curve) is compared to that obtained using the strong amplitudes $M_i^h$ only (dashed curve). The drastic modification of the function $F_1(\vartheta)$ at small $\vartheta$, as compared to the strong contribution, is due to interference. The pure electromagnetic contribution is negligible.

Let us consider the quantities $\sigma_{1s}$, $\sigma_{2s}$, $\sigma_{1d}$, and $\sigma_{2d}$ [see Eq. (12)], which determine the functions $\Omega_{+-}$ and $\Omega_{-+}$. The latter correspond to spin-flip transitions at $\vartheta \leq \theta_{acc} \ll 1$. We obtain for the small-angle asymptotics of the functions $G_X(\vartheta)$

$$G_{1s} = |M_2^h(0)|^2, \quad G_{2s} = \tfrac{1}{2}|M_2^h(0)|^2 + \tfrac{1}{2}|M_1^h(0) - M_3^h(0)|^2,$$

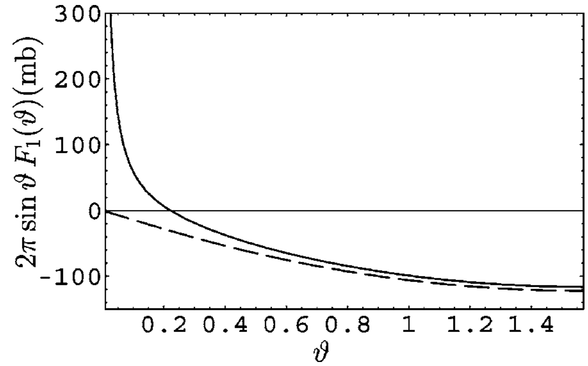


FIG. 1. The function $2\pi\sin\vartheta F_1(\vartheta)$ in units mb, calculated with the use of full amplitudes (solid curve) and of strong amplitudes only (dashed curve).

$$G_{1d} = -|M_2^h(0)|^2, \quad G_{2d} = \mathrm{Re}\{M_2^{h*}(0)[M_1^h(0) - M_3^h(0)]\}. \quad (20)$$

This contribution is mainly strong, and the corresponding cross sections can be estimated as $\sigma_s \sim \sigma_d \sim \theta_{acc}^2\sigma_1^h$, being negligibly small. The pure electromagnetic contribution to $\sigma_s$ and $\sigma_d$ as well as the interference of electromagnetic and strong amplitudes are negligible as compared to the strong contribution. In the nonrelativistic approximation, this follows from Eq. (15). If we take into account the first relativistic correction to the electromagnetic amplitudes (see, e.g., Ref. [9]), then the additional term appears in the functions $G_{1s}$ and $G_{2s}$,

$$\delta G_{1s} = 2\delta G_{2s} \simeq \frac{1}{8}\left(\frac{\alpha(4\kappa+3)}{m_p\vartheta}\right)^2, \quad (21)$$

where $\kappa=1.79$ is the anomalous magnetic moment of the proton in units of the nuclear magneton. Though the corresponding cross section is logarithmically enhanced, it is negligibly small being proportional to $\alpha^2/m_p^2$. Thus, scattering events, where projectiles stay in the beam, do not lead to beam polarization.

## IV. PURE ELECTRON TARGET

For scattering off electrons, the time dependence of the beam polarization is described again by Eq. (5), but with other expressions for the cross sections in Eqs. (11) and (12), which now correspond to the pure electromagnetic electron-proton interaction. The cross section accounting for the polarizations of the initial and final proton and electron is well known (see, e.g., Ref. [10]).

Here we consider head-on collisions of electrons and protons with momenta $p_e$ and $p_p$ in the laboratory frame. Assuming $\theta_{acc} \geq m_e/m_p \approx 0.5$ mrad, we obtain from kinematics that the particle loss takes place only for $p_e > p^{\mathrm{out}}$, where

$$p^{\mathrm{out}} = \frac{p_p\theta_{acc}m_p}{\varepsilon_p + p_p}, \quad (22)$$

$\varepsilon_p = \sqrt{p_p^2 + m_p^2}$ is the proton energy. For $p_e < p_p m_p/(\varepsilon_p + p_p + m_p)$, the maximal scattering angle $\theta_{\max}$ is

$$\theta_{\max} = \arcsin\left(\frac{\varepsilon_p p_e + \varepsilon_e p_p}{m_p(p_p - p_e)}\right), \qquad (23)$$

where $\varepsilon_e$ is the electron energy. In particular, $\theta_{\max}=m_e/m_p$ for $p_e=0$. For $p_e > p^{\text{out}}$, the cross sections $\sigma_0$, $\sigma_1$, and $\sigma_2$ entering in Eq. (11) $\Omega_{\pm}^{\text{out}}$, which are related to the filtering mechanism, can be estimated as

$$\sigma_0 \sim \frac{4\pi\alpha^2}{(p_p\theta_{\text{acc}})^2}\left(\frac{\varepsilon_e\varepsilon_p + p_e p_p}{\varepsilon_e p_p + \varepsilon_p p_e}\right)^2, \quad \frac{\sigma_1}{\sigma_0} \sim \frac{m_e m_p (p_p\theta_{\text{acc}})^2}{(\varepsilon_e\varepsilon_p + p_e p_p)^2},$$
$$\frac{\sigma_2}{\sigma_0} \sim \frac{(p_p\theta_{\text{acc}})^2}{\varepsilon_e\varepsilon_p + p_e p_p}. \qquad (24)$$

For the time $t\sim\Omega_{\text{tot}}^{-1}$, when $N(t)/N(0)$ is not too small [see Eq. (5)], the ratios $\sigma_1/\sigma_0$ and $\sigma_2/\sigma_0$ give the estimates of the beam polarization $P_B(t)$ for $\boldsymbol{\zeta}_T \perp \boldsymbol{\nu}$ and $\boldsymbol{\zeta}_T \parallel \boldsymbol{\nu}$, respectively. These ratios are maximal for $p_e \gtrsim p^{\text{out}}$, so that $P_B < m_e/m_p$ for $\boldsymbol{\zeta}_T \perp \boldsymbol{\nu}$, and $P_B < p_p\theta_{\text{acc}}/m_p$ for $\boldsymbol{\zeta}_T \parallel \boldsymbol{\nu}$. In both cases $P_B$is too small, and the mechanism using the loss of particles due to proton scattering off polarized electrons (filtering) does not work for any parameters of the electron beam.

Let us now consider the mechanism of the polarization buildup without loss of the particles, which is due to spin-flip transitions. Starting with the general expressions for the cross section of polarized electron-proton scattering [10], we find that the cross sections $\sigma_s$ and $\sigma_d$ in Eq. (12) are maximal at small relative velocity of electron and proton. In this case, we estimate $\sigma_d \sim 16\pi\alpha\mu_p^2 \sim 10^{-3}$ mb, $\mu_p$ is the proton magnetic moment. As compared to $\sigma_d$, the cross section $\sigma_s$ is enhanced by some logarithmic factor, which cannot change the conclusion on smallness of $\sigma_s$. Thus, for a pure electron target, the mechanism of polarization buildup based on spin-flip transitions does not work either.

We emphasize that the cross sections $\sigma_s$ and $\sigma_d$ are small because in spin-flip transitions there is no interference between the spin-dependent part of the amplitude and the spin-independent part of the Coulomb amplitude, which is proportional to $1/(v^2\vartheta^2)$ at $\vartheta \to 0$ ($v$ is the relative velocity). In processes without spin-flip transitions, which contribute to $\sigma_{1,2}$, such an interference is present. As a result the cross sections $\sigma_{1,2}$ turn out to be much larger than $\sigma_{s,d}$, $\sigma_{1,2}/\sigma_{s,d} \sim m_p/(m_e v^2) \gg 1$.

In conclusion, the kinetics of the polarization buildup in stored beams interacting with a polarized target has been investigated. It is demonstrated that, for $\theta < \theta_{\text{acc}}$ (a proton remains in the beam), the polarization buildup is completely due to spin-flip transitions. The corresponding cross sections turn out to be negligibly small for both proton-proton and proton-electron scattering. For a pure electron target, filtering mechanism does not provide a noticeable polarization either. Evidently, these statements are valid for the antiproton beam as well. Thus, the filtering method based on a hydrogen gas target with proton polarization seems to be the most promising way to polarize stored antiprotons. However, the spin-dependent parts of the proton-antiproton elastic and annihilation cross sections are not well known. Therefore, for stored antiprotons, further experimental and theoretical investigations are needed to obtain quantitative predictions for the time of polarization and for the polarization degree of the beam.

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